

The twisted wing:

(1)

In order to obtain aerodynamic force and moment distributions along the span, the wing is often twisted geometrically or aerodynamically or both.

(1) Geometric twist is achieved by twisting the axis of the wing so that the geometric angle of attack varies spanwise.

Geometric washout: α decreases from root to tip

(2). Aerodynamic twist is achieved by changing the airfoil section from root to tip, effecting a spanwise variation of camber and position of maximum camber. These variations affect the spanwise on the variation of the absolute angle of attack and center of pressure.

For aerodynamic washout could be achieved by decreasing the camber from root to tip.

For a given twist and planform in steady motion the absolute d_a and chord c are known. To determine numerically the spanwise distribution of sectional lift and drag, we select K spanwise stations $\theta_1, \theta_2, \dots, \theta_K$ at which wing chord c_1, c_2, \dots, c_K . Then after changing the upper limits of the summations from ∞ to K .

$$\frac{d_s}{\rho_{0j} C_s} \sum_{n=1}^K A_n \sin n\theta_j + \frac{\cos C_s}{4b} \sum_{n=1}^K n A_n \frac{\sin(n\theta_j)}{\sin \theta_j} = d_{aj} \quad (1)$$

In general for accuracy calculation, K is increased or by decreasing the relative spanwise interval.

$$Y_{j+1} - Y_j = \frac{b}{2} (\cos \theta_{j+1} - \cos \theta_j) \quad (2)$$

Properties will be changed by the sections such as near wing tips and in the vicinity of nacelles or engine pods.

The above equations will yield K simultaneous equations, which is solved for the coefficient A_1, \dots, A_K .

The local values of the effective and induced angles of attack at the j th station are.

$$d_{effj} = \frac{a \cos C_s}{a_{0j} C_j} \sum_{n=1}^K A_n \sin(n\theta_j) \quad (3)$$

$$d_{ij} = d_{effj} - d_{aj} = - \frac{\cos(C_s)}{4b} \sum_{n=1}^k n A_n \cdot \frac{\sin(n\theta_j)}{\sin\theta_i} \quad (4)$$

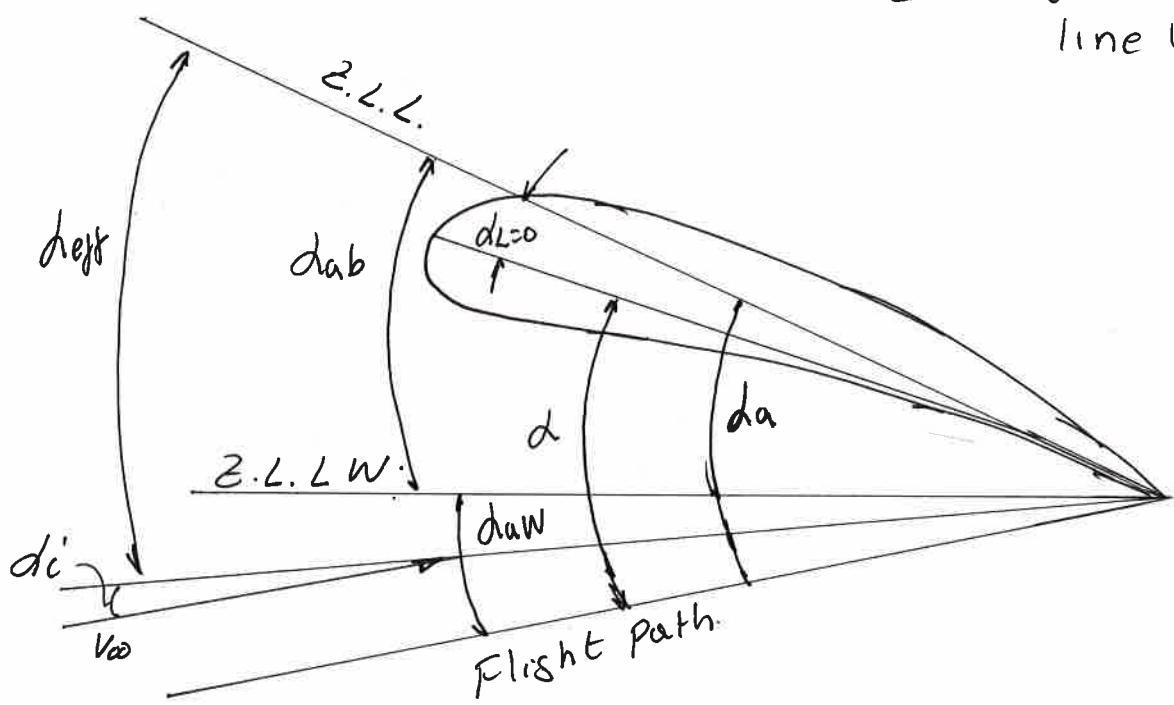
The local slope of the C_L vs. α_a curve is given by

$$d_{ij} = \frac{d_{0j}}{1 - d_{ij}/d_{effj}} \quad (5)$$

These equations gives the slope at each of the stations for the Δa distribution, represented by the equation of Δa_j , that is, for one given attitude of the wing relative to the flight path.

The above calculations outlines the procedure for finding the distributions of aerodynamic characteristics for a given δ (γ). We assume the wing is unstalled at every section.

Z.L.L.W (Zero lift
line wings)



In the above figure, angles are expressed relative to the direction designated Z.L.L.W., that is the direction of V_∞ for which the total lift of the wing is zero.

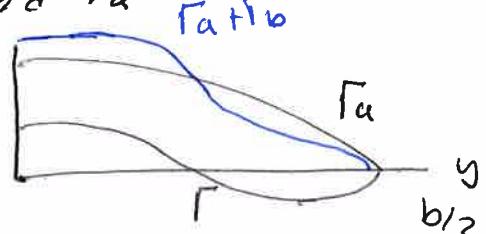
$$\Delta a = \alpha_{ab} + \alpha_{aw}$$

α_{ab} : basic absolute angle of attack, is the local absolute angle of attack for zero total lift of the wing and thus depends at a given station only on the wing twist.

α_{aw} : additional/absolute of attack, is measured from Z.L.L.W. to the flight path.

The corresponding spanwise circulation contributions designated, respectively Γ_b and Γ_a

$$\Gamma = \Gamma_a + \Gamma_b$$



they satisfy the relations

$$\int V_\infty \int_{-b/2}^{b/2} \Gamma_b dy = \int_{-b/2}^{b/2} L_b dy = 0$$

$$L = L_a + L_b = \int V_\infty (\Gamma_b + \Gamma_a)$$

corresponding lift coefficients are defined by

$$C_{Lb} = \frac{L_b}{g_\infty C} \quad C_{La} = \frac{L_a}{g_\infty C}$$

(3)

The local lift coefficient C_L is

$$C_L = C_{Lb} + C_a \quad (6)$$

where C_{Lb} depends on α_{ab} , that is, on the twist of the wing, and is thus independent of α_w , C_a (the additional lift coefficient) is dependent on α_w and is thus independent of wing twist.

$$C_L = C_{Lb} + C_a C_L \quad (7)$$

The equation (6) is solved for two angles of attack, that is, for two sets of α_{aj} designated α_{aj_1} and α_{aj_2} to obtain two sets of coefficients A_{11}, \dots, A_{1K_1} and A_{22}, \dots, A_{K_2} with the limitations of the lifting line representation equation (5).

$d_j = \frac{dC_{Lj}}{d\alpha_{aj}}$ is a constant of a given station, independent of α_a but varying station to station.

We approximate $\alpha_{j_1} = \alpha_{j_2} = \alpha_j$ and

$$C_{Lj_1} = \alpha_j \cdot d_{aj_1} \quad C_{Lj_2} = \alpha_j \cdot d_{aj_2} \quad (8)$$

If these coefficients are integrated spanwise with respect to y , we obtain

$$C_{L_1} = \frac{1}{S} \int_{-b/2}^{b/2} C_{L_1} \cdot c \, dy = \frac{\alpha \cos C_s \pi b}{4S} A_{11} \quad (8)$$

$$C_{L_2} = \frac{1}{S} \int_{-b/2}^{b/2} C_{L_2} \cdot c \, dy = \frac{\alpha \cos C_s \pi b}{4S} A_{12}$$

Then we write equations (7) for two angle of attack

$$C_{L_{1j}} = C_{Lbj} + C_{La j} C_{L_1} \quad (9a)$$

$$C_{L_{2j}} = C_{Lbj} + C_{La j} C_{L_2}$$

Substitute (8) and (9). The solution of simultaneous equations (10) yield that C_{Lb} and C_{La} as functions of α . The equations can be solved for these coefficients and C_L versus α . Determined throughout the range of angle of attack.

The spanwise distribution of induced drag can be found

$$C_{di} = -C_L d_i \quad (11) \quad \text{and} \quad d_i = \delta_{eff} - \alpha = \frac{C_L}{\alpha_0} - \frac{C_L}{\alpha}$$

so that

$$C_{di} = C_L^2 \left(\frac{\alpha_0 - \alpha}{\alpha_0 \alpha} \right) \quad (12)$$

We may also define a "weighted mean slope" (9)

$$\bar{\alpha} = \frac{1}{S} \int_{-b/2}^{b/2} ac \cdot dy \quad (13)$$

so that

$$C_L = \bar{\alpha} \cdot \text{law.} \quad (14)$$

EXAMPLE 6.4

This example demonstrates the use of the analysis for computing the performance of a twisted wing of finite span. The symmetrical trapezoidal wing shown in Fig. 6.16 is considered, which has the following properties:

Aspect ratio: $AR = 6$.

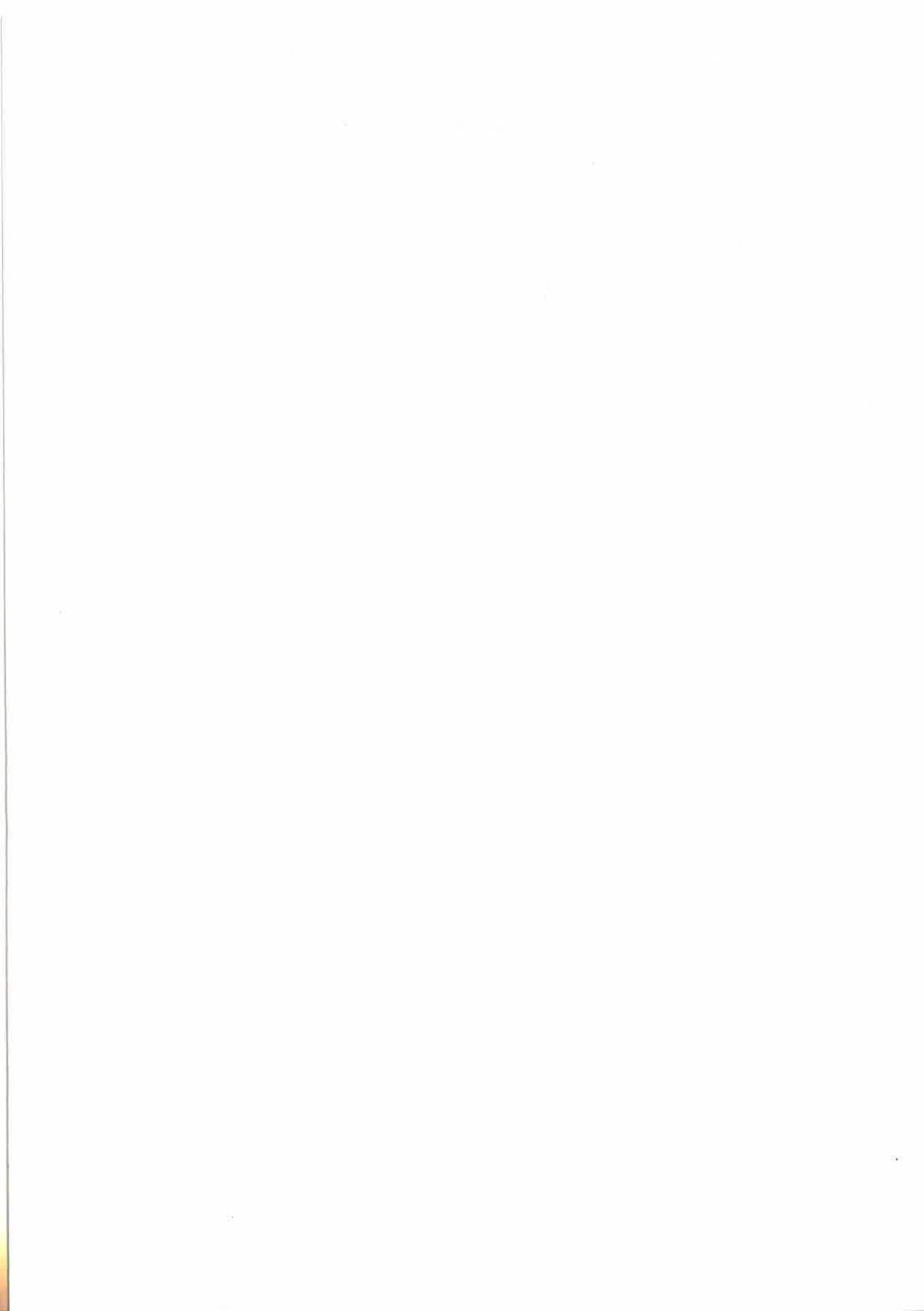
Taper ratio: $\lambda = (\text{tip chord } c_t)/(\text{root chord } c_s) = 0.55$.

The wing has a geometric twist that varies linearly from zero at the root to -4° at the tip.

A negative twist denotes washout.

There is no aerodynamic twist.

Airfoil shapes are identical and m_0 has a constant value of 2π per radian along the span.



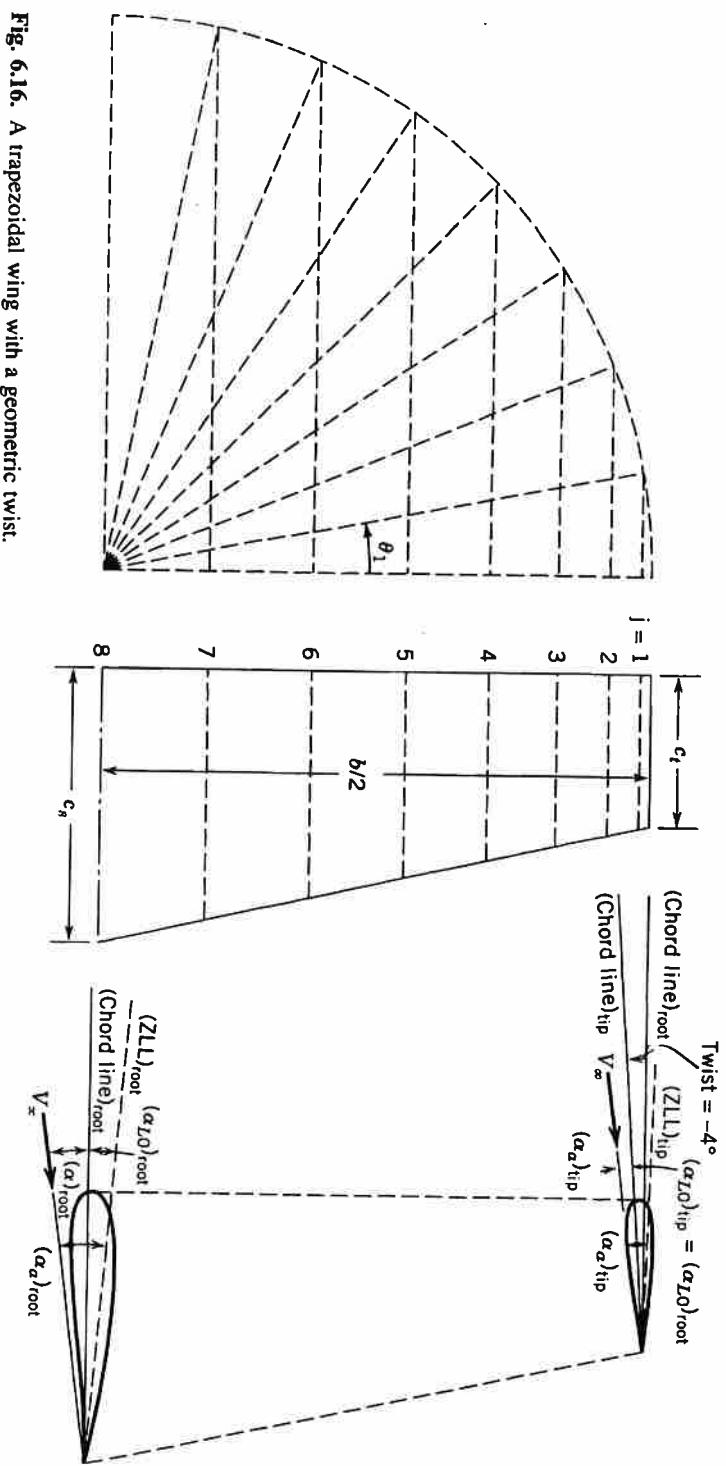


Fig. 6.16. A trapezoidal wing with a geometric twist.

- (a) Compute sectional aerodynamic properties c_{l_0} and c'_{l_0} for this wing.
 (b) When flying at sea level at speed V of 250 km/hr, the wing loading W/S is 800 N/m². Compute c_l and c_{d_l} along the span and the wing characteristics for this flight condition.

As shown in Fig. 6.16, eight spanwise stations are selected on one side of the wing, so that $k = 8$ in Eq. (6.40). We defined dimensionless spanwise distance Y as $y/0.5b$ and dimensionless chord C as c/c_s . At the j th station on a linearly tapered wing,

$$\theta_j = \frac{\pi j}{2k}; \quad Y_j = \cos \theta_j; \quad \text{and} \quad C_j = 1 - (1-\lambda) \cos \theta_j$$

It can also be verified that

$$\frac{m_0 c_s}{4b} = \frac{\pi}{AR(1+\lambda)}$$

Because the coefficients A_n vanish for even values of n for a symmetrical wing, it is more convenient to replace A_n by A_N in Eq. (6.40) and to replace n by $2N - 1$ elsewhere in that equation, where $N = 1, 2, \dots, k$. Thus, Eq. (6.40) can be written in a simplified form

$$\sum_{N=1}^k D_{jN} A_N = \alpha_{a_j}; \quad j = 1, 2, \dots, k \quad (6.40a)$$

in which, for a uniform distribution of $m_0 = 2\pi$,

$$D_{jN} = \left[\frac{1}{C_j} + \frac{(2N-1)\pi}{AR(1+\lambda) \sin \theta_j} \right] \sin(2N-1)\theta_j$$

D_{jN} is a function of geometry only and can readily be computed for given values of j and N .

Similarly, Eqs. (6.35), (6.41), (6.42), and (6.53) are rewritten as

$$C_{l_i} = \frac{\pi^3}{AR(1+\lambda)^2} \sum_{N=1}^k (2N-1) A_N^2 \quad (6.35a)$$

$$c_{l_j} = \frac{2\pi}{C_j} \sum_{N=1}^k A_N \sin(2N-1)\theta_j \quad (6.41a)$$

$$\alpha_{ij} = -\frac{\pi}{AR(1+\lambda)} \sum_{N=1}^k (2N-1) A_N \frac{\sin(2N-1)\theta_j}{\sin \theta_j} \quad (6.42a)$$

$$\bar{m} = \frac{1}{1+\lambda} \int_0^1 m C dY$$

The integral in the last equation can be evaluated numerically using the trapezoidal rule:

$$\bar{m} = \frac{1}{2(1+\lambda)} \sum_{j=1}^{k-1} (m_j C_j + m_{j+1} C_{j+1}) (Y_j - Y_{j+1}) \quad (6.53a)$$

$$\begin{aligned}
 A_1 &= 0.0425 & A_5 &= 0.0006 \\
 A_2 &= -0.0036 & A_6 &= -0.0003 \\
 A_3 &= 0.0027 & A_7 &= 0.0003 \\
 A_4 &= -0.0008 & A_8 &= -0.0002
 \end{aligned}$$

$$V = 250 \text{ Km/hr} \quad \text{Wing Loading} = 800 \text{ N/m}^2$$

$$\begin{aligned}
 C_{Dc} &= \frac{\pi^3}{6(1+0.55)^2} \left[(0.0425)^2 + 3(0.0036)^2 + 5 \cdot (0.0027)^2 \right. \\
 &\quad + 7 \cdot (0.0008)^2 + 9 \cdot (0.0006)^2 \\
 &\quad + 11 \cdot (0.0003)^2 + 13 \cdot (0.0003)^2 \\
 &\quad \left. + 15 \cdot (0.0002)^2 \right]
 \end{aligned}$$

$$\left[C_{Dc} = 0.00435 \right] : \cancel{C_{p1} = 0.00027}$$

$$\left[C_L = 0.269 \right. = \frac{V^2 / m^2}{\rho g} = \frac{800 \text{ N/m}^2}{1 \times 1.23 \times (69.44)^2}$$

$$C_{Dc} = \frac{(0.018)^2 (1+0.021)}{\pi \times 6}$$

$$\left[\cancel{C_{p1} = 0.00027} \right] = \frac{C_{Dc}}{0.00435} = 0.00386$$

$$C_{Lw} = 0.271$$